# Safe Minimum Headway on a Single Track (revised 10/14/2019)

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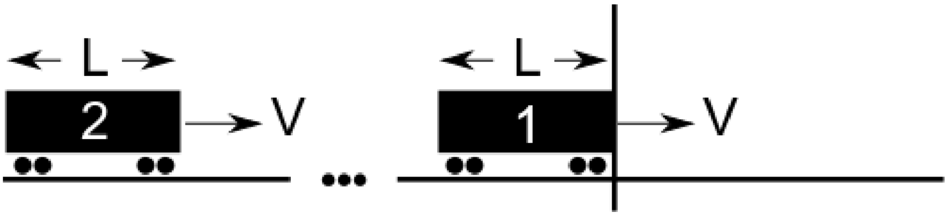
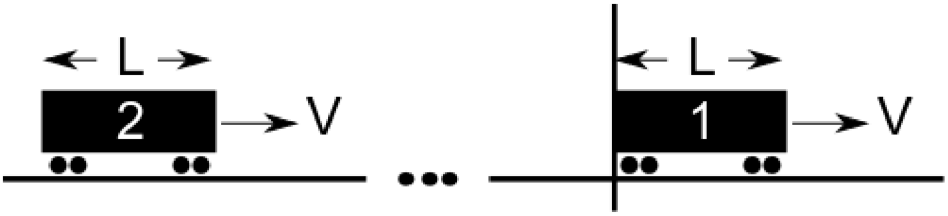
## Abstract.

**Simple Case – No Stations.** The analysis for determining the minimum headway between trains on a single track can be shown in Drawing 1, below. This drawing shows two trains, traveling from left to right. The rightmost train is numbered 1 and is the leader. The leftmost train is numbered 2 and is the follower. This snapshot is taken when the leader is approaching a the zero point and the follower is an arbitrary but safe distance behind the leader.



*Drawing 1: Leader and Follower at time $t = 0$.*

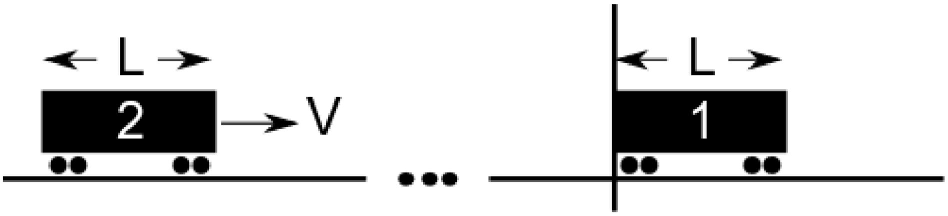
Both the leader an follower continue traveling at velocity V, as shown in Drawing 2, below.



*Drawing 2: Leader and Follower at time $t = T_1$.*

The leader has just passed the zero point the zero point, shown as the vertical line, in Drawing 2. If the leader's length is L and its velocity is V, then then the duration for the leader to travel across the zero point will be L/V. Thus, $T_1 = L/V$.

At this instant, the leader suddenly stops but the follower continues at velocity V at a safe distance behind the leader. The follower is unaware that the leader has stopped. This



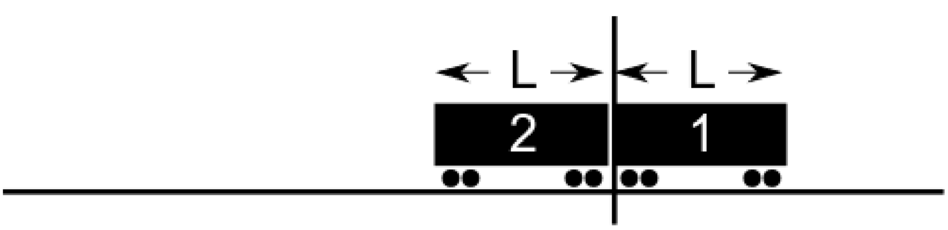
*Drawing 3: Leader and Follower at time $t = T_1 + \delta$*

snapshot is shown in Drawing 3, above. The leader's stopping time, $\delta$, is an arbitrarily small amount of time that approaches 0.

The signal system alerts the follower that the leader has stopped. It takes the signal system a finite time, $T_S$, to determine that the leader has stopped and alert the follower. This time is based on the communications timeout, for CBTC systems. It is based on the follower's distance from the block behind the leader and that block's length, for conventional block systems. Drawing 4, above, shows the leader and follower at the instant the signal system has notified the follower that the leader has stopped.

*Drawing 4: Leader and Follower at time $T = T_1 + \delta + T_S$.*

The follower immediately applies the emergency brakes. If the follower was at a safe distance, it will stop just short of the leader. Drawing 5, above, shows the instant where the follower stops just short of the leader. The time $T_{EB}$ is the followers emergency braking time from velocity V. It is equal to $V/a_{eb}$, where $a_{eb}$ is the emergency braking rate.



*Drawing 5: Leader and Follower at time $T = T_1 + \delta + T_S + T_{EB}$.*

Substituting the values for $T_1$ and $T_{EB}$ from above the headway, T becomes:

$$T=\frac{L}{V}+\delta+T_S+\frac{V}{a_{EB}} \quad \text{eq 1.}$$

When $\delta$ is set to 0, equation 1 reduces to:

$$T=\frac{L}{V}+T_S+\frac{V}{a_{EB}} \quad \text{eq 2.}$$

This relation is shown in Illustration 1, on page 3. It shows two sets of values for $a_{EB}$ and $T_S$. The value of 3.0 mph/sec for $a_{EB}$ and 0 for $T_S$, are typical for block systems. The value of 1.4 mph/sec for $a_{EB}$ and 2.0 sec for $T_S$ are typical for CBTC systems.

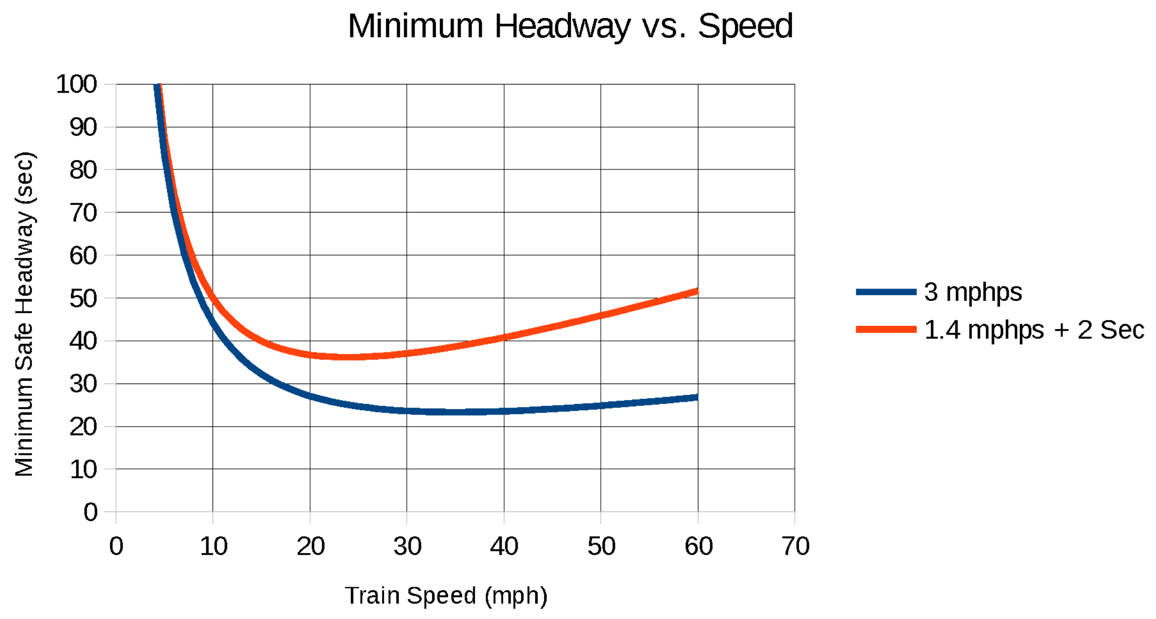


*Illustration 1: Minimum Headway vs. Train Speed.*

Dividing the minimum headway into 3600 yields the service level in trains per hour (TPH). This data is shown in Illustration 2, below.

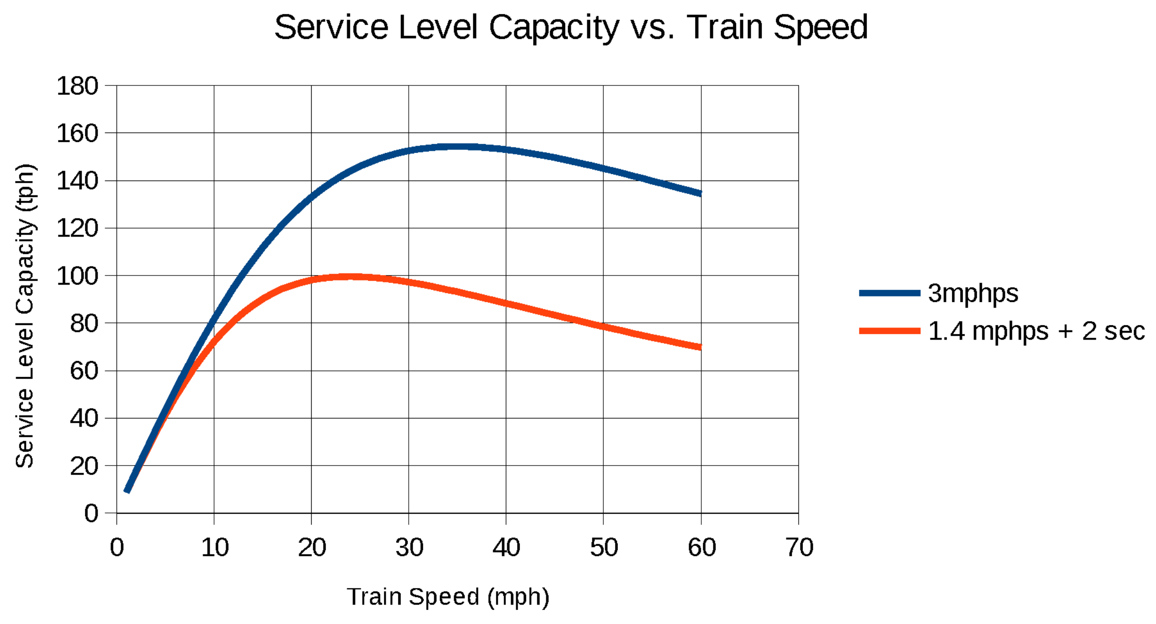
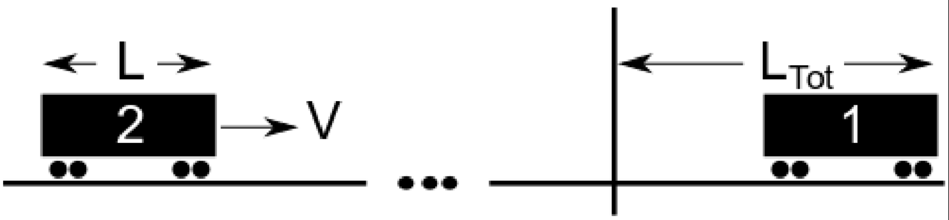


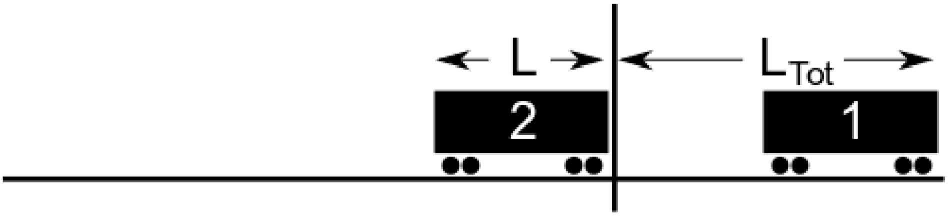
*Illustration 2: Service Level Capacity vs. Train Speed*

A block system has uncertainty as to the leader's exact location. This is shown in Drawing 6, below. This drawing is similar to Drawing 3, on page 1. The difference is that the leader has traveled a different distance, $L_{Tot}$, before coming to a sudden stop. The time, $T_1$, for the leader to travel this distance is give by $T_1 = L_{Tot}/V$.



*Drawing 6: Leader and Follower at time $t = T_1 + \delta$, with uncertainty in leader's position.*

Similarly, the follower will stop on the mark as shown in Drawing 7, below. The signal delay time, $T_S$, is essentially 0 for a conventional block system because there is no communications delay.



*Drawing 7: Leader and Follower at time $T = T_1 + \delta + T_S + T_{EB}$, with uncertainty in leader's position.*

Assuming both $T_S$ and $\delta$ are zero, the relation shown in Drawing 7 reduces to:

$$T = T_1 + T_{EB} = \frac{L_{Tot}}{V} + \frac{V}{a_{EB}} \quad \text{eq. 3}$$

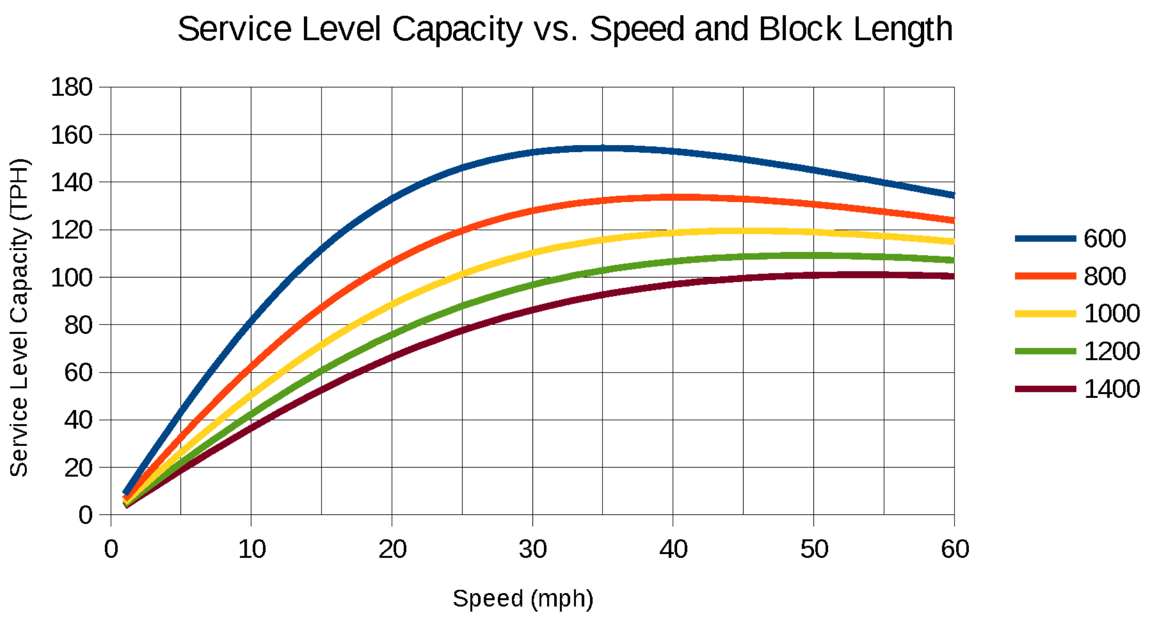


*Illustration 3: Service Level Capacity vs. Speed for Various Block Lengths*

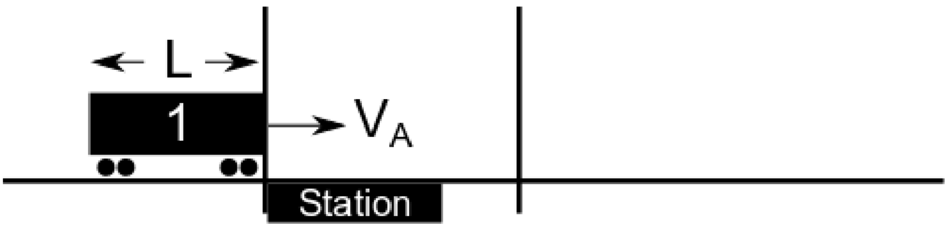
The service level capacity is the inverse of the minimum safe headway, shown in equation 3. It is shown in Illustration 3, for various block lengths, and an emergency braking rate of 3 mphps.

Illustration 3 shows that service level varies inversely with block length and train speed. This relationship can be used to reduce the block count for a desired service level capacity. A service level capacity of 100 tph requires a block length of 1400 feet at 50 mph. Should track conditions require a speed reduction to 25 mph, e.g. downgrade or curve, then the service level capacity would be reduced to 80 tph with 1400 foot blocks. However, if the block length on this section were reduced to 1000 feet, then the 100 tph service level could be maintained.

**<u>Stations</u>.** The presence of stations increases the safe minimum headway and service level capacity. This can be analyzed by breaking the scenario into 9 discrete parts: 1 - the leader approaches the station entrance traveling at a speed $V_A$; 2 - the leader continues into the station at the same speed, VA; 3 – the leader brakes at a constant rate and stops at the station's exit; 4 – the leader stops within the station (dwell time); 5 – the train accelerates at a constant rate to a departure traveling speed $V_D$; 6 – the leader suddenly stops in zero time; 7 – the follower is traveling at an minimum distance behind the leader at the leader's traveling speed, $V_D$; 8 – the follower becomes aware that the leader has stopped after the signal systems delay time $T_S$; 9 – the follower applies the emergency brakes and stops at a distance $L_{TOT}$ behind the leader's front.

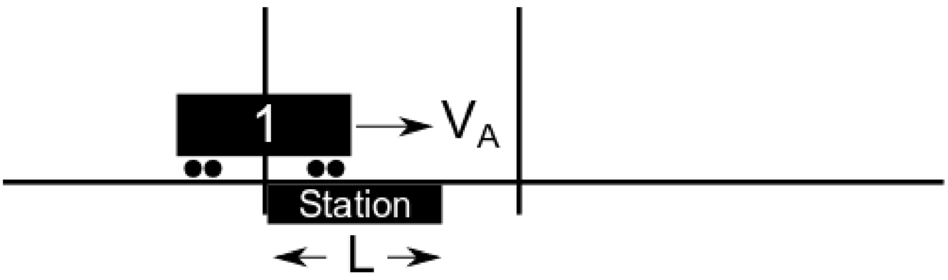
The times for each part can be derived. Their total represents the minimum safe headway.

Part 1 is the leader traveling at approach speed at the station entrance. This time snapshot is shown in Drawing 8, below.



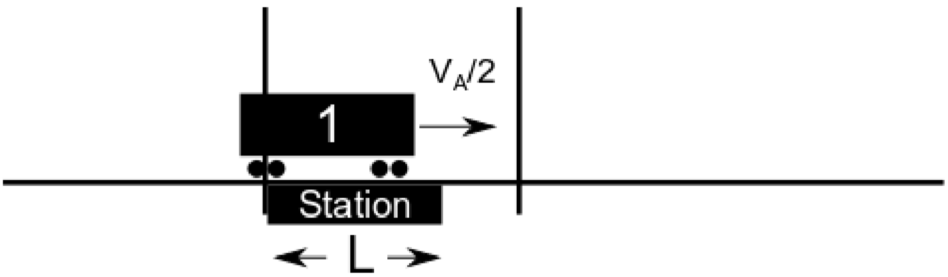
*Drawing 8: Part 1 - leader entering station at traveling speed.*

Part 2 is the leader entering the station at approach speed. This time snapshot is show in Drawing 9, below.



*Drawing 9: Part 2 - leader continues at traveling speed into station before braking.*

Part 3 is the leader slowing down into the station under a uniform service braking rate $a_S$. The average speed is half the initial speed, $V_A$. This time snapshot is shown in Drawing 10, below. The time to stop from the initial velocity, $V_A$, to a complete stop is: $V_A/a_S$. The distance traveled while braking is: $V_A^2/2a_S$.



*Drawing 10: Part 3 - leader braking within station.*

This quantity permits the calculation of the distance and time traveled at $V_A$ before the brakes were applied. The distance traveled is: $L - V_A^2/2a_S$. The leader maintained a speed of $V_A$ so the time was $(L/V_A) – (V_A/2a_S)$.

The total time from when the train enters the station at speed until it stops is given by the sum of the time spent at $V_A$ and the braking time. Its value is:

$$T_{Stop}=\frac{L}{V_A}+\frac{V_A}{2a_S} \qquad \text{eq 4.}$$

This function is shown in Illustration 4, for a train length, L, of 600 feet and a service braking rate of 3 mphps (4.41 fpsps). This value is easy to measure. An observer, with a stop watch, at the station entrance measures the elapsed time between a train's entrance until it comes to a complete stop. The nominal value for this time is 30 seconds.

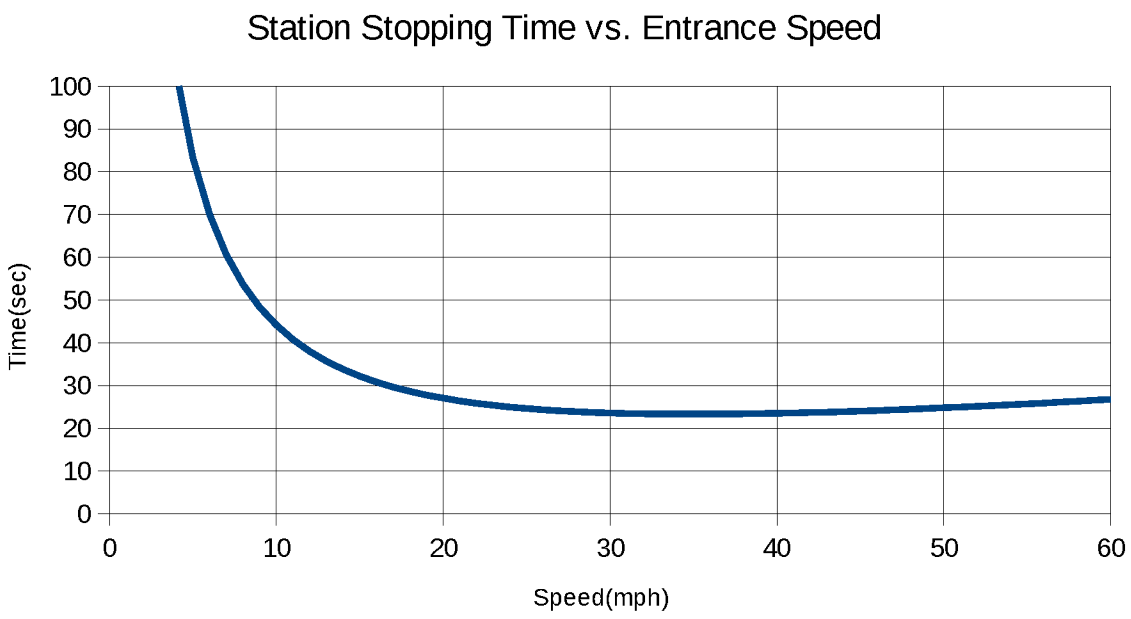
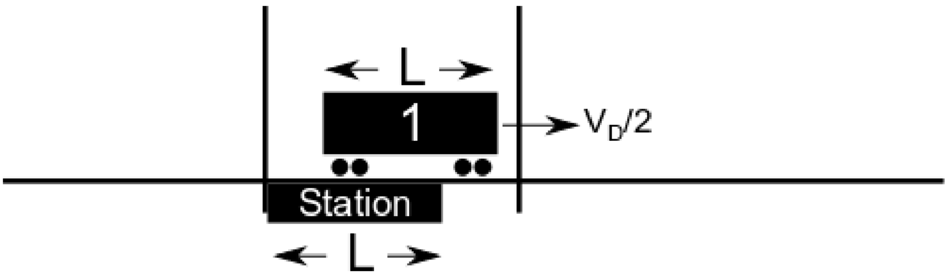


*Illustration 4: Station Stopping Time vs. Entrance Speed*

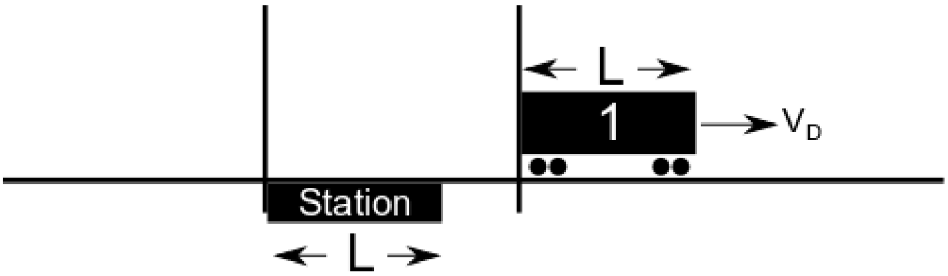
Part 4 is the station dwell time. The dwell time consists of: 1 - the train stopped in the station with the doors closed waiting for the doors to open; 2 – the time stopped in the station with the doors open and passengers crossing the door threshold; 3 – the time stopped in the station with the doors open but no passengers crossing the door threshold; 4 - the time stopped in the station with the doors closed waiting for the train to start moving. Most literature places the first component with the braking time and limits the dwell time to components 2, 3, and 4.

Part 5 is the train’s acceleration time. The train travels from 0 to $V_D$, at a constant acceleration $a_D$. The time the train to accomplish this is $V_D/a_D$.



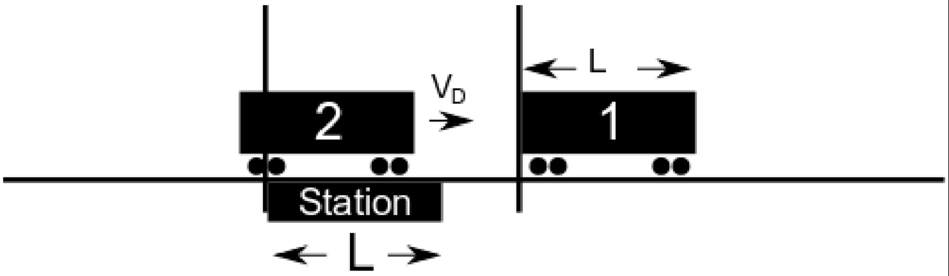
*Drawing 11: Part 5 - Train starting to move out of at constant acceleration.*

Part 6 is when the train will reach $V_D$, when its rear reaches the right hand vertical line as shown in Drawing 12, below. The time for the train to reach traveling speed $V_D$ from rest is: $V_D/a_D$, where $a_D$ is the train’s acceleration.



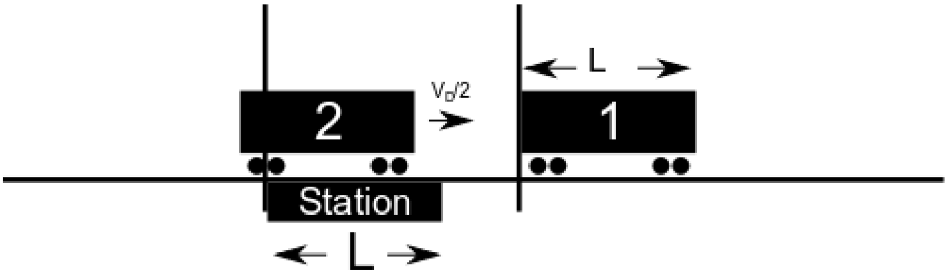
*Drawing 12: Part 6 - Train reaches traveling speed.*

Part 7 is when the train suddenly stops due to something unforseen. There is a follower traveling at the same speed that is the minimum safe stopping distance behind. This is illustrated in drawing 13, below.



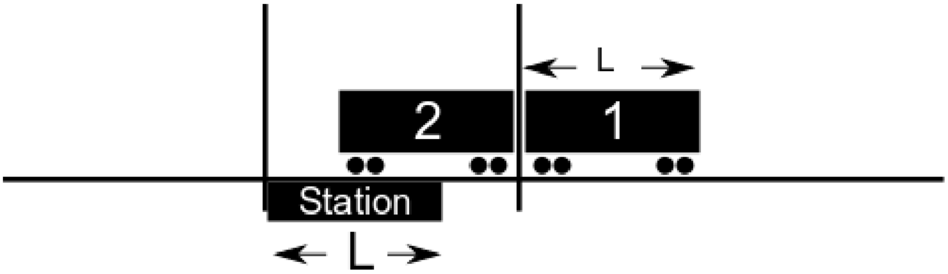
*Drawing 13: Part 7 - Train suddenly stops, follower traveling at minimum stopping distance at speed $V_D$.*

Part 8 is when the follower senses that the leader has stalled and brakes at the emergency braking rate. This is shown in Drawing 14, below. Ideally, there is no delay in detecting that the leader has stalled. Practically, there are delays for both CBTC and block systems. The CBTC delay consists of the communications delay, as well as uncertainty as to the leader's position. The block system delay consists of the follower's travel time to reach the empty block behind leader.



*Drawing 14: Part 8 - Follower is braking at emergency braking rate.*

Part 9 is when the follower comes to a complete stop, just behind the leader. The time for the follower to come to this complete stop is: $V_D/a_{EB}$, where $a_{EB}$ is the emergency braking rate. Drawing 15, below shows the follower stopping just short of the stalled leader.



*Drawing 15: Part 9 - Follower comes to safe stop.*

The time from when the leader starts moving from the station until the follower safely stops behind the stalled leader is given by:

$$\frac{V_D}{a_D}+\frac{V_D}{a_{EB}}=\left(\frac{a_D+a_{EB}}{a_D a_{EB}}\right)V_D \quad \text{eq 5.}$$

This function is shown in illustration 5, below, assuming values of 2.5 mphps and 3.0 mphps for $a_D$ and $a_{EB}$, respectively.

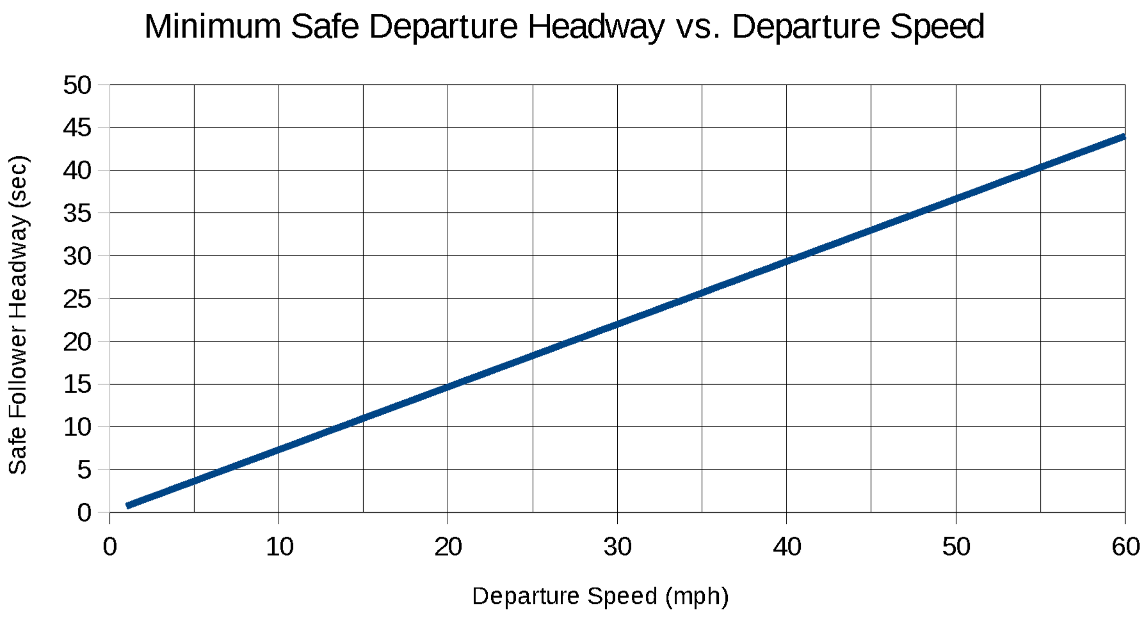


*Illustration 5: Minimum Safe Departure Headway vs. Departure Speed, assuming 2.5 mph/sec acceleration and 3.0 mp/sec emergency braking rate.*

This illustrates that the nominal departure time is 30 seconds for a nominal departure speed of 40 mph and no signal system delays. Assuming a fixed 2.0 second signal system delay, the nominal 30 second departure time is valid for 37 mph or less.

Combining equations 4 and 5 and adding in the station dwell time, the overall minimum safe headway becomes:

$$T_{Headway}=\frac{L}{V_A}+\frac{V_A}{2a_S}+\left(\frac{a_D+a_{EB}}{a_D a_{EB}}\right)V_D+T_{Dwell}+T_{Signal} \quad \text{eq 6.}$$

Under a perfect signal system, $L_{Tot} = L$, and $T_{Signal} = 0$. This means that there is no signal delay for a CBTC system and the block lengths are infinitesimally small.

Illustration 6, below, shows the simple case, where $V_A = V_D = V$, $a_S = 3$ mphps, $a_{EB} = 3$ mphps, $a_D = 2.5$ mphps, $L = 600$ ft, $T_{Dwell} = 45$ sec.

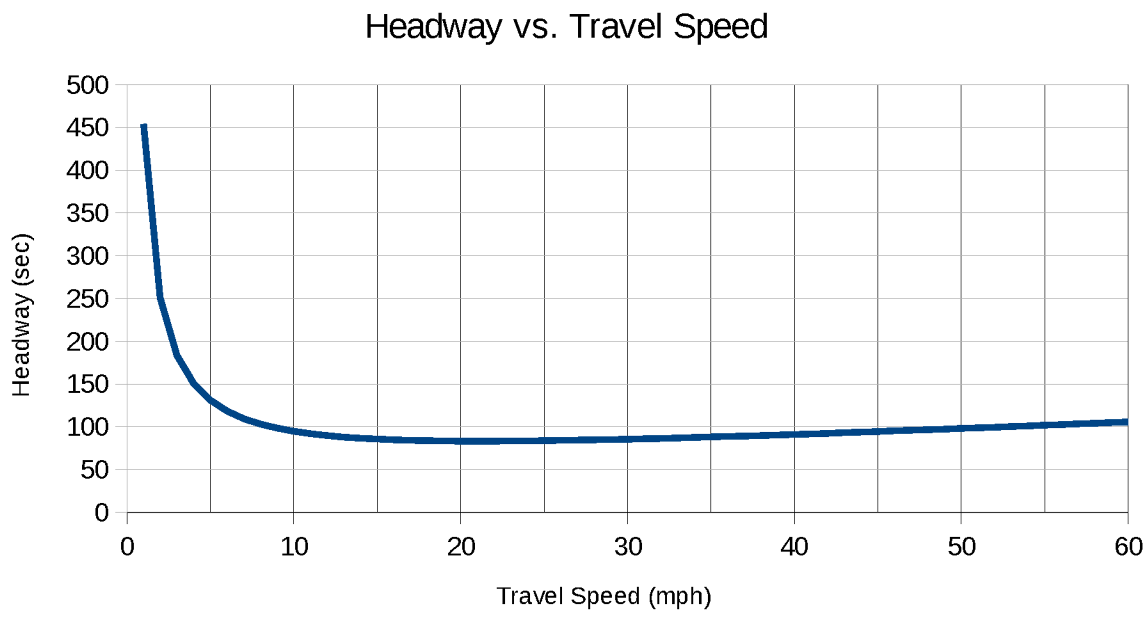


*Illustration 6: Minimum Safe Headway vs. Train Speed for realistic parameters.*

Illustration 7, below shows the equivalent service level capacity.

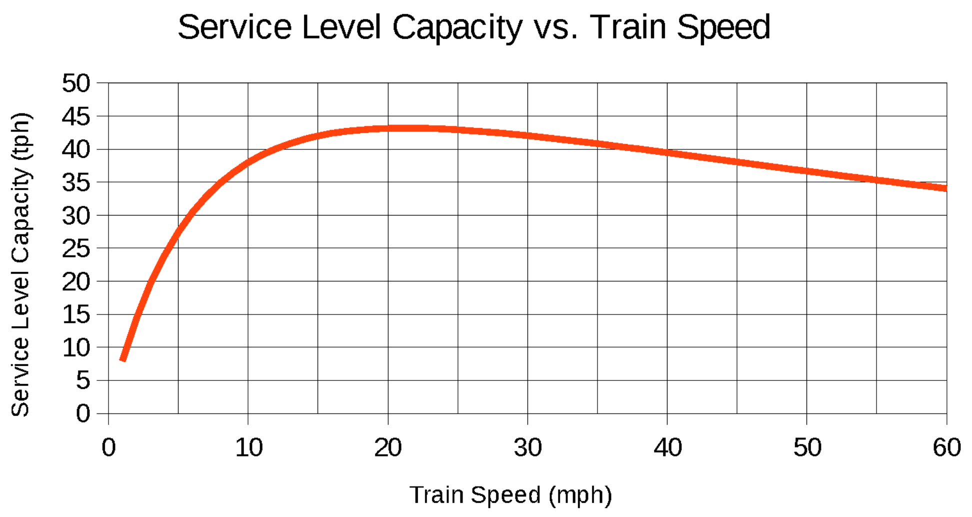


*Illustration 7: Equivalent service level capacity for minimum safe headway in illustration 6.*

This illustration confirms the nominal 40 tph (90 second headway) limitation for intermediate stations regardless of the signal system.